# Introducing Prime Space

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**Abstract**

This paper suggests that primes are not numbers but dimensions, which allow all natural numbers to be generated. If the notion of a dimension is defined as an independent extension which cannot be reduced, primes are reasonable candidates. A differentiation between primes and natural numbers is proposed, which turns primes into prerequisites for the generation of natural numbers.
The author presents this approach as a suggestion to reinterpret the relation between primes and (positive) natural numbers. Primes are entangled with many mathematical theorems and they also have pratical applications. It is beyond the scope of this paper, however, to apply the suggested differentiation between primes and natural numbers to these theorems or applications. This has to be adressed elsewhere. The intention of this paper is limited to providing a differentiated way of looking at primes.

## Introduction

Primes are usually treated as a subset of the set of natural numbers. The only property which differentiates them from natural numbers is a constraint on their divisibility: Primes can only be divided by 1 or themselves. However, if the notion of a dimension is taken seriously, primes should be regarded not as numbers but as dimensions which allow all natural numbers to be generated, as every natural number may be expressed as a product of prime numbers.

The concept of a dimension has been defined as "one of a group of properties whose number is necessary and sufficient to determine uniquely each element of a system of usually mathematical entities (as an aggregate of points in real or abstract space" [1]. A dimension may also be defined as an independent, non-reducible extension. Against the background of these definitions, primes are reasonable candidates for dimensions.

## Prime Space

If primes are regarded as dimensions, they generate a multidimensional space, in which the individual axes represent units made up of primes. Every natural number may be represented as a point defined by all the axes necessary to host the primes used in the factorization of this number.

For each natural number or set of natural numbers, a multidimensional prime space may be set up, which consists of only those primes (=dimensions) which are included in the factorization of that number or that set of numbers. A more complex space is also conceivable, which is made up of all primes up to an arbitrary limit. This continuous Prime Space could represent a range of natural numbers.

The geometrical representation of Prime Space is limited. Two and three dimensions may be represented by the axes in a Cartesian co-ordination system. These axes do not necessarily have to represent the primes 1,2,3, but may stand for any other combination, e.g. 3,5,11 to represent the number 165. If more than 3 primes are needed for factorization, however, Prime Space can no longer be represented graphically. From here on, analytic geometry has to take over.

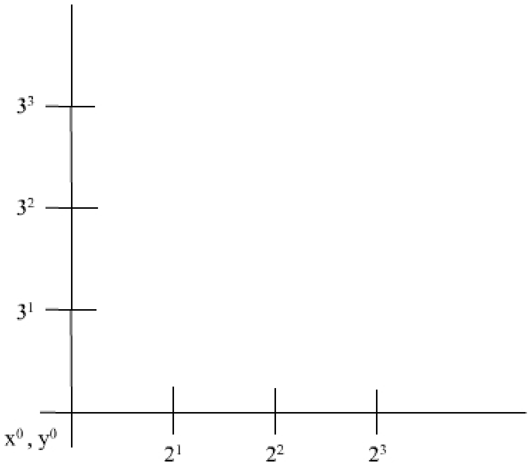


Figure 1

As exemplified in Figure 1, for two axes (x and y, representing factorizations of 2 and 3), the origin is shared by all axes, as $2^0 = 0$, $3^0 = 0$, $5^0 = 0$, etc.
The axes are divided into units of e.g. $3^0$, $3^1$, $3^2$, $3^3$, $3^4$, ...., with $3^0$ being situated at the origin.

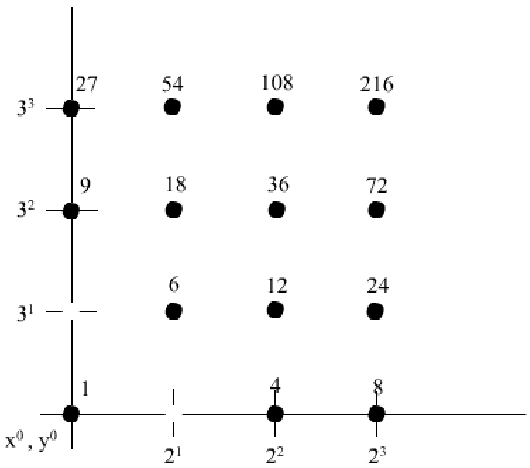


Figure 2

Figure 2 shows the set of natural numbers generated by the factorization of the primes 2 and 3. An extention to this representation may be achieved by adding a z-axis, which would host, for example, factorizations of the prime 5. But this is where a graphical representation meets its limit. Further dimensions would have to be expressed in an algebraic formulation, which allows a more generalized definition: A subset of Prime Space, i.e. a natural number may be defined by its prime co-ordinates:

Example 1: the element {165} is a natural number which lives in Prime Space (3,5,11).

Example 2: the element {495} is also a natural number which lives in Prime Space (3, 5, 11). The only difference to the preceding example is that here, the prime 3 is raised to the power of 2.

**Why differentiate between natural numbers and primes?**

In Prime Space, primes are still treated as the "atomic" building blocks of the set of natural numbers. They are, however, no longer numbers themselves, as they cannot be represented in multidimensional Prime Space themselves – they are the dimensions which render possible the generation natural numbers (this is why they are not represented in Figure 2).
The new aspect this approach entails is the option of differentiating between numbers and primes on a more basic level. So far, primes have been distinguished from natural numbers only by their property of being divisible only by themselves and 1. They are still, however, regarded as subsets of the natural numbers. The approach suggested in this paper introduces a Prime Space within which all natural numbers may be portrayed as a set derived from the a priori dimensional dependencies of the primes required for their factorization. (NB: primes are not a subset of the set of natural numbers in the new model!)

If primes and natural numbers are differentiated on this fundamental level, i.e. treating primes as dimensions and other integers as natural numbers, then they are each assigned a new status: The natural numbers now presuppose the existence of primes, as they can only be generated and portrayed in Prime Space. Natural numbers live in Prime Space. Primes do not.

Again, it is a logical differentiation which prompts the approach presented here. To present an analogy, chemical elements in Mendeleev‘s periodic table would correlate to the set of primes, if the defining abstraction is the lack of divisors.
If one tried to reconstruct the periodic table analogous to the set of natural numbers, we would have to fit in all possible molecules which may result from combinations of the foregoing elements with the following one in-between the respective elements. The result would mirror – in the field of chemistry – the way we believe in and deal with the set of natural numbers. Although the fields may be incommensurable, it makes one wonder whether treating primes as natural numbers is elegant or logical. If one takes the concept of a dimension seriously, primes should not appear on the same level of description as natural numbers, just as elements should not be listed in one table with molecules.

**Primes portrayed in chryzodes**

Apart from this logical distinction between dimensions and numbers, Prime Space changes the portrayal of natural numbers, as primes are no longer part of the set of natural numbers. All natural numbers and their relations can be portrayed as a point in Prime Space – either geometrically in a Cartesian coordination system (if three prime dimensions suffice for the factorization of a particular natural number) or as a chryzode (which may host many more constraints than the axes of a Cartesian coordination system). [2]

A chryzode is a graphical representation within a circle which is calibrated as a clock, whose "hours" divide the circumference of the circle at equidistant points into intervals of equal lengths. A 5-hour clock, for example, would consist of 5 equidistant points on the circumference of the circle. Then paths linking the hours of this 5-hour clock are inserted. Figure 3 shows a superposition of a 2- , 3- , 5-, 7- and 11-hour clock, including the paths linking the individual clocks. The intersection points represent interactions between the various elements of the clocks.

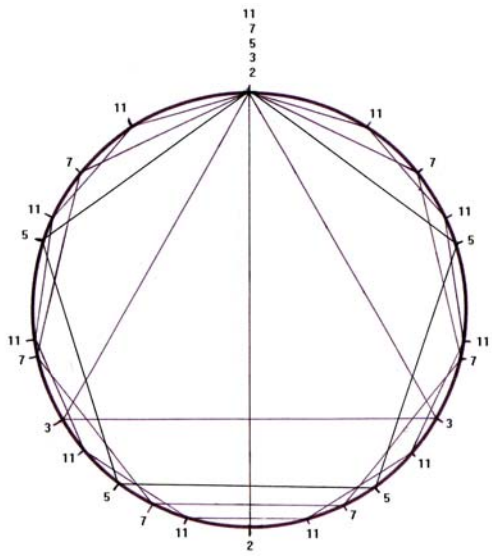

Figure 3

Chryzodes may host, theoretically, an infinite number of primes as constraints (i.e. points on the circumference of a circle, which divide the circle into equal intervals). If superimposed, the geometrical representation portrays an attractor inside the circle. This, in turn, allows us to see regularities which cannot be derived from other forms of representation. If the individual steps are animated, this renders possible an intuitive understanding of the dynamics of the generation of natural numbers (represented by the intersections). Each natural number thus occurs on a number of intersections (see Figure 4).

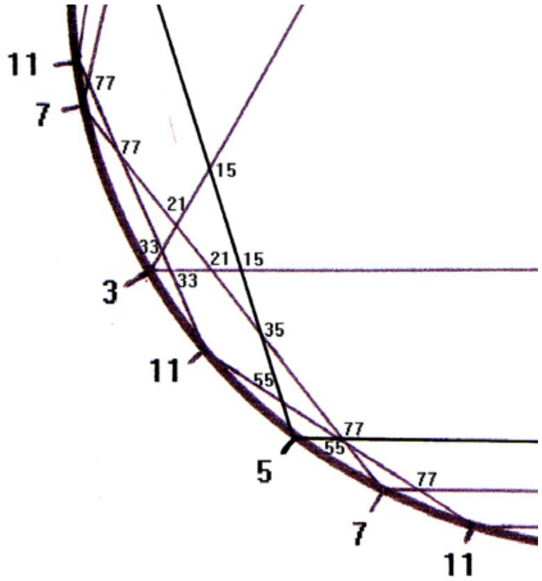


Figure 4

Chryzodes may reveal properties which are hidden from view in an open graphical representation such as a Cartesian co-ordination system. For the set of natural numbers (which do not include primes), the number of occurrences of the natural numbers within the circles shows an underlying ordering principle. This ordering principle is a result of the closedness of chryzode space.
Thus, the attractor in this Chryzode Prime Space reveals an additional property of natural numbers as a result of their generation, namely their "specific weight". This property may only be portrayed in a closed representation.
The Chryzode Prime Space is closed, as the superimposed primes act as constraints. This graphical representation creates redundancy, as each natural number (apart from the smaller numbers, such as 6 and 8, as well as multiples of one prime number) is generated several times at the points of intersection.

**Conclusion**

Questions concerning an exploitation of the differentiation of natural numbers in terms of their"specific weight" and the high redundancy created for large numbers must be debated eleswhere. Here, it shall suffice to present an epistemological differentiation between primes and natural numbers. This differentiation is a logical step if one takes the notion of a dimension seriously.
Further research into the various areas affected is necessary to show whether or not the suggested logical distinction will make a useful difference in the application of primes.

**References**

[1] Merriam-Webster Online Dictionary, http:/www.m-w.com/cgi-bin/dictionary?book=Dictionary&va=dimension

[2] cf. www.chryzode.org